SEISMIC RESPONSE OF EQUIPMENT LOCATED WITHIN ASYMMETRIC BUILDING STRUCTURES

Magdi F. Ishac and Arthur C. Heidebrecht

SYNOPSIS

The purpose of this paper is to describe the mathematical formulation used in computing the seismic response of equipment located within asymmetric buildings and to illustrate the effect of lateral-torsional coupling of this particular class of structures on the equipment response. The equipment response is represented by floor response spectra.

For the coupled analysis of asymmetric structures, the parameters of interest are the lateral floor acceleration and the rotational floor acceleration. Each floor motion time history is used as input to a series of damped single degree of freedom systems in order to determine the lateral and rotational floor response spectra. The response results are analyzed to consider the influence of the lateral-torsional coupling of the structure on the equipment response.

RESUME

Dans cet article, on présente les équations mathématiques utilisées pour calculer la réponse aux séismes de l'équipement d'un bâtiment dissymétrique et on étudie l'effet de la torsion sur le comportement sismique de cet équipement.

Pour l'analyse des structures dissymétriques, les paramètres importants sont l'accélération horizontale et l'accélération angulaire des planchers. La variation dans le temps des déplacements de chaque plancher est utilisée comme donnée d'entrée pour l'étude du comportement d'une série de systèmes à un degré de liberté avec amortisseurs. Cette étude permet de déterminer les spectres de réponse des planchers pour les translations latérales et les rotations. Avec ces résultats on peut étudier l'influence de la torsion de la charpente sur la réponse aux séismes de l'équipement d'un bâtiment.

Magdi F. Ishac obtained his B.Sc.(Eng.) from Cairo University in 1972, his M.Eng. from McMaster University in 1976, and is a doctoral candidate at McMaster University. He is currently employed as a design engineer with Ontario Hydro in Toronto.

Arthur C. Heidebrecht obtained his Ph.D. from Northwestern University in 1963, following a B.Sc. in Civil Engineering from the University of Alberta in 1960. He is a Professor of Civil Engineering at McMaster University in Hamilton and is currently Chairman of the Canadian National Committee on Earthquake Engineering.

INTRODUCTION

Earthquake motions induce inertial forces in all parts of a structural system, including the soil underlying the structure and the structure contents including mechanical and electrical systems. The response of structures, equipment and piping to such dynamic motions is quite complex.

It is usually impractical to include such equipment in the dynamic model representing the building because of the large difference between the mass of the equipment and that of the building. Therefore, the equipment and the building are treated separately and the building response is used as input for the equipment analysis. For obtaining the structural response, ground motion is applied at the base of the structure. The resulting motion of the structure is similarly imparted to the equipment to determine its response. The term used to define the equipment response is the "Floor Response Spectrum".

A floor response spectrum is a graphical display of the maximum responses of a family of single-degree-of-freedom oscillators mounted on a particular floor of a structure which is subjected to seismic ground motion; these maximum responses are computed for a particular level of damping and for a range of natural periods (e.g. 0.01 - 10.0 sec).

For asymmetric building structures, and due to lateral-torsional coupling, the resulting motion of the structure consists of lateral response and rotational response. Due to the double input excitations applied to the equipment, a lateral floor response spectrum and a rotational floor response spectrum are needed.

For some types of equipment, the effect of the rotational input motion may have a significant contribution. However, the major factor which may affect the equipment response is its location, not only its elevation within the structure, but also its lateral location relative to the centre of the building.

SEISMIC RESPONSE OF ASYMMETRIC BUILDING STRUCTURE

The mathematical formulation used for the seismic response of uniform tall building structures, is developed for the case of one axis of symmetry. This particular class of building structure includes those whose lateral load resisting system is made up of an asymmetrically arranged grouping of flexure type (shear walls) and shear type (frames) elements. These flexural beam and shear beam elements are assumed to be relatively uniform[1] with respect to height and combined by using the equilibrium and compatibility conditions to yield a coupled set of partial differential equations (2).

The equations of motion governing the case of seismic ground motion $\ddot{y}_G(t)$ applied to the entire structure can be written in the form

$$\begin{bmatrix} \frac{1}{b^2} & 0 \\ 0 & \frac{1}{c^2} \end{bmatrix} \begin{bmatrix} \ddot{y}_c \\ r\ddot{\theta} \end{bmatrix} + \left[\begin{array}{c|c} \frac{\partial^4}{\partial z^4} - \alpha^2 \frac{\partial^2}{\partial z^2} & \frac{e_m}{r} \frac{\partial^4}{\partial z^4} - \frac{a_m}{r} \alpha^2 \frac{\partial^2}{\partial z^2} \\ \hline \frac{e_m}{r} \frac{\partial^4}{\partial z^4} - \frac{a_m}{r} \alpha^2 \frac{\partial^2}{\partial z^2} & \frac{\partial^4}{\partial z^4} - \beta^2 \frac{\partial^2}{\partial z^2} \end{array}\right] \begin{bmatrix} y_c \\ r\theta \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{b^2} & 0 \\ 0 & \frac{1}{c^2} \end{bmatrix} \begin{bmatrix} \ddot{y}_G(t) \\ 0 \end{bmatrix} \qquad (1)$$

(The notations are given in the glossary.)

The two coupled displacements may be assumed to take the form

$$\begin{bmatrix} y_c\ (z,t) \\ r\theta\ (z,t) \end{bmatrix} = \sum_{i=1}^{\infty} T_i(t) \begin{bmatrix} \phi_{yi}\ (z) \\ \phi_{\theta i}\ (z) \end{bmatrix} \qquad (2)$$

in which $T_i(t)$ represents the variation with time (t) for the ith mode of vibration and $[\phi_{yi}(z)\ \phi_{\theta i}(z)]$ represent the coupled mode shapes for this mode of vibration.

---

[1]For non-uniform structural-system the determination of the specific eccentricity and stiffness parameters becomes more complex, but parameters equivalent to these for the uniform case exist and the formulation proceeds in very much the same manner.

The eigenvalues and eigenvectors can be determined for the case of free vibration by using numerical procedures already available in the literature.

Rearranging the left hand side of Eq. (1) by substituting Eq. (2), premultiplying the resulting equation for the ith mode of vibration by $[\phi_{yi}(z)\ \phi_{\theta i}(z)]$, integrating along the total height of the structure and using the orthogonality-normality relationship given by

$$\int_0^H \left[\frac{(\phi_{yi})^2}{b^2} + \frac{(\phi_{\theta i})^2}{c^2}\right] dz = 1 \tag{3}$$

the damped form of Eq. (1) can be written as

$$\ddot{T}_i(t) + 2\zeta\omega_i\dot{T}_i + \omega_i^2 T(t) = \Gamma_{yi}\ \ddot{y}_G(t) \tag{4}$$

in which Γ_{yi} is the modal participation factor for mode i

$$\Gamma_{yi} = \int_0^H \left(\frac{\phi_{yi}}{b^2}\right) dz \tag{5}$$

Eq. (4) can be solved by numerical integration (1) to determine the response of the ith mode; the total responses can then be superimposed to determine the overall system response at any floor level of the structure by using Eq. (2).

In order to be able to evaluate the effect of coupling between lateral and torsional response, it is necessary to determine the uncoupled response as well. In the uncoupled case, the effect of eccentricities in Eq. (1) is ignored and the uncoupled equations of motion take the form:

$$\frac{1}{b^2}\ddot{y}_u + \left(\frac{\partial^4}{\partial z^4} - \alpha^2\frac{\partial^2}{\partial z^2}\right) y_u = \frac{1}{b^2}\ \ddot{y}_G(t) \tag{6.a}$$

$$\frac{1}{c^2}\, r\ddot{\theta} + \left(\frac{\partial^4}{\partial z^4} - \beta^2\frac{\partial^2}{\partial z^2}\right) r\theta = 0 \tag{6.b}$$

The eigenvalue problems become

$$\left(\frac{d^4\psi_y}{dz^4} - \alpha^2\frac{d^2\psi_y}{dz^2}\right) - \frac{\omega_y^2}{b^2}\psi_y = 0 \tag{7.a}$$

$$\left(\frac{d^4\psi_\theta}{dz^4} - \beta^2\frac{d^2\psi_\theta}{dz^2}\right) - \frac{\omega_\theta^2}{c^2}\psi_\theta = 0 \tag{7.b}$$

and lead to the uncoupled vibration frequencies ω_{yi}, $\omega_{\theta i}$ and the uncoupled lateral and rotational mode shapes $\psi_{yi}(z)$ and $\psi_{\theta i}(z)$.

For the uncoupled response analysis, the modal participation factors used are determined by

$$\Gamma_i = \int_0^H \left(\frac{\psi_{yi}}{b^2}\right) dz \tag{8}$$

and the lateral floor acceleration response $\ddot{y}_u$ is obtained using the same procedure described above. The rotational response vanishes in this case and it can be generated only if a torsional ground motion is applied to the structure, this situation is not within the scope of this study.

The modal response factors associated with the response parameters of interest are shown in Table 1.

FLOOR RESPONSE SPECTRA

The significance of a general building analysis is then to provide dynamic inputs to the internal equipment. Once an analytical model of the building structure has been developed, such a model can be used to determine the input motions to equipment and other secondary systems supported within the building. The term used to define this environment in a generalized form is the "Floor Response Spectrum", which is specified for a particular range of frequencies and value of equipment damping.

There are several methods available to determine floor response spectra. The most popular method to generate floor response spectra is the time history method; it is the most straight forward one as far as the theory is concerned. The basic assumption used in the time history approach is that the mass of the equipment is so small in relation to that of the structure such that there is no feed-back from the equipment to the structure.

To determine the floor response spectra by the time history approach or by any simplified method, it is usual to consider planar models of the structure in each of the two orthogonal directions and to independently analyze the response of each model to the in-plane horizontal component of earthquake ground motion. None of these methods considers the torsional response of the building structure and its effect on the floor response spectra generated. Since earthquake motions occur randomly and not necessarily along the orthogonal axes of building structures, some torsional response may be induced in symmetrical or nearly symmetrical buildings as well as in asymmetrical structures. This fact is recognized in the National Building Code of Canada (7), where a nominal level of accidental torsion must be considered for symmetrical structures. Of course, when a structure is not symmetrical, a torsional analysis must be done. Because torsional response of the structure may modify floor response spectra values significantly, especially at the extreme edge, it is the objective of this study to investigate the effect of torsional coupling on floor response spectra. To illustrate the effect of such coupling the following four cases are considered:

a) FRS - $\ddot{y}_u$: Uncoupled floor response spectrum

b) FRS - $\ddot{y}_c$: Centroidal coupled floor response spectrum

c) FRS - $\ddot{y}_{e+}$: (+ve) Edge floor response spectrum

d) FRS - $\ddot{y}_{e-}$: (-ve) Edge floor response spectrum

The uncoupled floor motion ($\ddot{y}_u(t)$) is obtained from the uncoupled analysis. In the coupled analysis, both lateral and rotational floor motions ($\ddot{y}_c(t)$, $\ddot{\theta}(t)$) are generated, with the rotational component arising due to the lateral-torsional coupling within the building structure. The extreme edge floor motions ($\ddot{y}_{e+}(t)$, $\ddot{y}_{e-}(t)$) are developed by

$$\ddot{y}_{e+}(t) = \ddot{y}_c(t) + d.\ddot{\theta}(t) \tag{9.a}$$

$$\ddot{y}_{e-}(t) = \ddot{y}_c(t) - d.\ddot{\theta}(t) \tag{9.b}$$

in which d is the horizontal distance from the center of mass to the extreme edge of the building.

The rotational floor response spectra can be obtained by applying the rotational floor motion to a series of torsional single degree of freedom oscillators and plotting their maximum rotational responses as a function of their natural period for a particular level of damping.

NUMERICAL EXAMPLE

The floor plan given in Figure 1 is that for a sixteen storey building. The storey height is 3.0 m. Taking the z-axis at the center of mass gives $e_m = -4.95$ m and $a_m = 0.0$. The basic parameters associated with the dynamic properties of the structure are

$EI_y = 14.68 \times 10^7$ kN.m^2 $\qquad$ $EI_\omega = 534.60 \times 10^7$ kN.m^4

$GA_y = 10.18 \times 10^4$ kN $\qquad$ $GJ = 8633.00 \times 10^4$ kN.m^2

$\rho A = 62.6$ kN.sec^2/m^2 $\qquad$ $\rho I_m = 2087.0$ kN.sec^2

Figure 2 shows the coupled flexural-torsional mode shapes for the first six normal modes. The natural periods and the modal response factors associated with the response parameters of interest are tabulated in Table 2.

Using the 1940 El Centro W-E earthquake record, whose ground response spectrum is shown in Figure 3, as input ground motion and assuming a constant damping factor of 0.05, the dynamic response of the example wall-frame building structure is computed incorporating the first six normal modes. The resulting lateral and rotational floor motions ($\ddot{y}_c(t)$, $\ddot{\theta}(t)$) are obtained at the top and at the mid-height of the building. The edge lateral floor motion ($\ddot{y}_{e+}(t)$, $\ddot{y}_{e-}(t)$) are generated to include the effect of torsional response. The uncoupled lateral floor motions ($\ddot{y}_u(t)$) are also determined at the top and at the midheight of the example structure. For each case, floor response spectra are generated using the floor motion time histories as input. The equipment or secondary damping used is assumed to be one percent, since such equipment is normally lightly damped. Figures 4 and 5 show the four lateral floor response spectra at the top and at the midheight of the building, respectively. The rotational floor response spectra are shown in Figure 6.

Comparing the uncoupled floor response spectra (FRS-$\ddot{y}_u$) to the centroidal coupled spectra (FRS-$\ddot{y}_c$), considerable deviations are observed. In general, the uncoupled ordinates exceed the centroidal coupled ones with few exceptions especially in the frequency range associated with the sixth normal mode of the building structure. The variation of the floor spectra ordinates is due to the variation of the modal response factors ($\Gamma_i\psi_{yi}\rightarrow\Gamma_{yi}\phi_{yi}$) and the period shift of the two models.

Considering the rotational floor response spectra, the illustration of torsional coupling can be shown. The peaks of these floor spectra are associated with the periods contributing large values of rotational modal response factors ($\Gamma_{yi}\phi_{\theta i}$), i.e. strong modal coupling effect. The rotational floor response spectra may express the frequency range affected by torsional coupling and also act as dynamic input for some types of equipment which can respond to a rotational input motion.

Comparing the lateral floor spectra generated at different locations of the floor level for the coupled lateral-torsional model (FRS-$\ddot{y}_c$, FRS-$\ddot{y}_{e+}$ and FRS-$\ddot{y}_{e-}$), it is clear that the torsional coupling may have major significance on the lateral floor spectra. The equipment response varies not only with respect to its elevation within the structure, but also with its lateral location relative to the center of the building. The variation of the floor spectra ordinates is due to the contribution of the rotational modal response factors ($\Gamma_{yi}\phi_{\theta i}$) induced as a result of torsional coupling.

Table 3 shows the maximum amplification factors (equipment to ground) associated with this particular example structure at different lateral location of the top and the midheight of the building. These amplification factors are obtained for rigid and flexible equipment. In this example, the term "flexible" applies to flexibly-mounted rigid equipment as well as to rigidly-mounted flexible equipment with a period of 0.03 seconds or greater. This period has been used since evaluation of ground response spectra generally show little or no amplification of seismic motion for periods less than 0.03 seconds.

The results of this example structure indicate that the following observations can be made:

a. All coupled centroidal amplification factors are less than the uncoupled values.

b. The effect of torsion produces larger coupled amplification factors at positive edge, which is expected due to the geometry of the building structure used in this example.

c. The rigid equipment amplification values are much lower than the flexible values because structural response does not produce significantly amplified floor motions for periods less than 0.03 seconds.

d. For flexible equipment, the largest coupled values are substantially less than the uncoupled values; in this case coupling has reduced the lateral response of equipment.

e. For rigid equipment, the largest coupled values are slightly larger than the uncoupled values; in this case the rotational effect has produced an increase in amplification.

Considering the National Building Code of Canada (7), the current edition (1977) uses amplification factors of 2, but it should be noted that the structure on which the equipment is mounted would be expected to respond beyond yield level which would reduce responses compared to the elastic case. Also, it should be noted that this value of amplification will be increased to 10 for the 1980 NBCC. However, if structures behave elastically, then the example structure shows that the amplification factors can be considerably larger. More work needs to be done in order to evaluate the amplification factors.

CONCLUSIONS

The results of this investigation indicate that the following conclusions and recommendations can be drawn:

1. Torsional coupling induces rotational motion which may have a significant contribution on the equipment response. The major factor which may affect the equipment response is its lateral location relative to the center of the building. If torsional effects are of major significance it may be advisable to develop extreme floor spectra enveloping all locations for any particular floor level.

2. The centroidal coupled floor spectra peaks are usually smaller than the corresponding uncoupled values with some exceptions due to a building frequency shift.

3. The effect of torsion produces larger values of the positive edge floor response spectra, which one would expect due to the geometry of the building structure used in the numerical example.

4. The results of this investigation are based on a very limited set of data; the validity for more general situations will require substantial application of this method of analysis to a large number of structures and input excitations in order to review the amplification factors suggested by the NBCC.

5. More investigations need to be made in order to determine guidelines to define situations for which the torsional coupling effect must be considered in the seismic analysis.

6. The concept of the rotational floor response spectrum can be applied in the same manner as the lateral floor response spectrum.

7. The proposed rotational floor response spectrum is a result of the lateral-torsional coupling of the structure due to the eccentricity between its center of mass and of rigidity.

REFERENCES

1. Ishac, M.F., "Dynamic Response of Asymmetric Shear Wall-Frame Building Structures". Thesis submitted in partial fulfillment of the requirements for the degree of Master of Engineering, McMaster University, Hamilton, Ontario, Canada, December, 1975.

2. Ishac, M., and Heidebrecht, A., "Dynamic Response of Asymmetric Shear Wall-Frame Building Structures". Proceedings Sixth World Conference on Earthquake Engineering, New Delhi, Vol. 3, January, 1977, pp. 3-373 to 3-378.

3. Nigam, N.C., and Jennings, P.C., "Programme SPECEQ, a Digital Calculation of Response spectra from Strong Motion Earthquake Records". NISEE Computer Applications, California Institute of Technology, Pasadena, California, June, 1968.

4. Ishac, M.F., "Torsional Coupling in Seismic Response of Reactor System". Ph.D. Thesis, to be submitted, McMaster University, Hamilton, Ontario, Canada, 1979.

5. Heidebrecht, A.C., and Stafford Smith, B., "Approximate Analysis of Tall Wall-Frame Structures". Journal of the Structural Division, ASCE, Vol. 99, No. ST2, February, 1973, pp. 199-221.

6. Heidebrecht, A.C., and Stafford Smith, B., "Approximate Analysis of Open-Section Shear Walls Subject to Torsional Loading". Journal of the Structural Division, ASCE, Vol. 99, No. ST12, December, 1973, pp. 2355-2373.

7. National Building Code of Canada 1977, National Research Council of Canada, Ottawa, Ontario, Canada.

GLOSSARY

a_m, e_m	shear and flexure component eccentricities, respectively, measured from the center of mass (m)
b^2 $(=\frac{EI_y}{\rho A})$, $c^2 (=\frac{EI_\omega}{\rho I_m})$	parameters describing stiffness and mass relationships in y and θ directions, respectively, (m^4/sec^2)
d	horizontal distance from the center of mass to the extreme edge
EI_y	translational flexure stiffness $(kN.\ m^2)$
EI_ω	warping torsional stiffness $(kN.\ m^4)$
GA_y	translational shear stiffness (kN)
GJ	St. Venant torsional stiffness $(kN.\ m^2)$
H	height of building, (m)
$r\ (=\sqrt{\frac{EI_\omega}{EI_y}})$	flexural radius of gyration, (m)
t	time variable (sec)
y_c	coupled lateral displacement of the center of mass (m)
y_u	uncoupled lateral displacement of the center of mass (m)
$\ddot{y}_G(t)$	input ground motion (m/sec^2)
z	height variable, (m)

$\alpha^2 \ (= \frac{GA_y}{EI_y}),\ \beta^2 \ (= \frac{GJ}{EI_\omega})$	parameters describing stiffness relationships in the y and θ directions, respectively, (m^{-2})
$\phi_{yi},\ \phi_{\theta i}$	coupled lateral and torsional shape functions for the ith mode of vibrations, respectively
ρA	mass per unit height of the building ($kN.\ sec^2/m^2$)
ρI_m	mass moment of inertia per unit height of the building ($kN.\ sec^2$)
θ	rotation about the center of mass (rad.)
ω_i	the ith natural frequency of the free coupled vibration (rad/sec)
$\omega_{yi},\ \omega_{\theta i}$	the ith uncoupled frequencies for the lateral and rotational motion, respectively
$\psi_{yi},\ \psi_{\theta i}$	uncoupled lateral and rotational mode shapes, respectively
ζ_e	equipment damping (1%)
ζ_s	structural damping (5%)

Table 1: Modal Response Factors

Response Parameters	Modal Response Factors
$\ddot{y}_c$	$\Gamma_{yi}\ \phi_{yi}$
$r\ddot{\theta}$	$\Gamma_{yi}\ \phi_{\theta i}$
$\ddot{y}_u$	$\Gamma_i\ \psi_{yi}$

Table 2: Natural Periods and Modal Response Factors - (Uncoupled Model versus Coupled Model)

	Uncoupled Model			Coupled Model				
Mode	Periods (sec)	Modal Response Factors		Periods (sec)	Modal Response Factors			
		Top	Midheight		Top		Midheight	
		$\Gamma_i \psi_{yi}$	$\Gamma_i \psi_{yi}$		$\Gamma_{yi}\phi_{yi}$	$\Gamma_{yi}\phi_{\theta i}$	$\Gamma_{yi}\phi_{yi}$	$\Gamma_{yi}\phi_{\theta i}$
1	2.143	1.545	0.563	2.220	1.509	0.126	0.585	0.089
2*	0.778	--	--	0.828	0.016	0.035	0.006	0.023
3	0.408	0.844	0.591	0.465	0.665	0.382	0.524	0.081
4*	0.228	--	--	0.245	0.050	0.020	0.013	0.025
5	0.151	0.507	0.005	0.200	0.134	0.465	0.027	0.259
6*	0.111	--	--	0.132	0.168	0.101	0.108	0.081

* torsional mode of vibration

Table 3: Maximum Amplification Factors
(Equipment to Ground)

Equipment	Top				Midheight			
	Uncoupled Model	Coupled Model			Uncoupled Model	Coupled Model		
		Cent-roidal	(+ve) Edge	(-ve) Edge		Cent-roidal	(+ve) Edge	(-ve) Edge
Rigid	3.9	3.0	4.7	2.9	1.7	1.5	2.2	1.5
Flexible	30.4	14.7	23.8	20.0	19.3	9.2	14.4	8.0

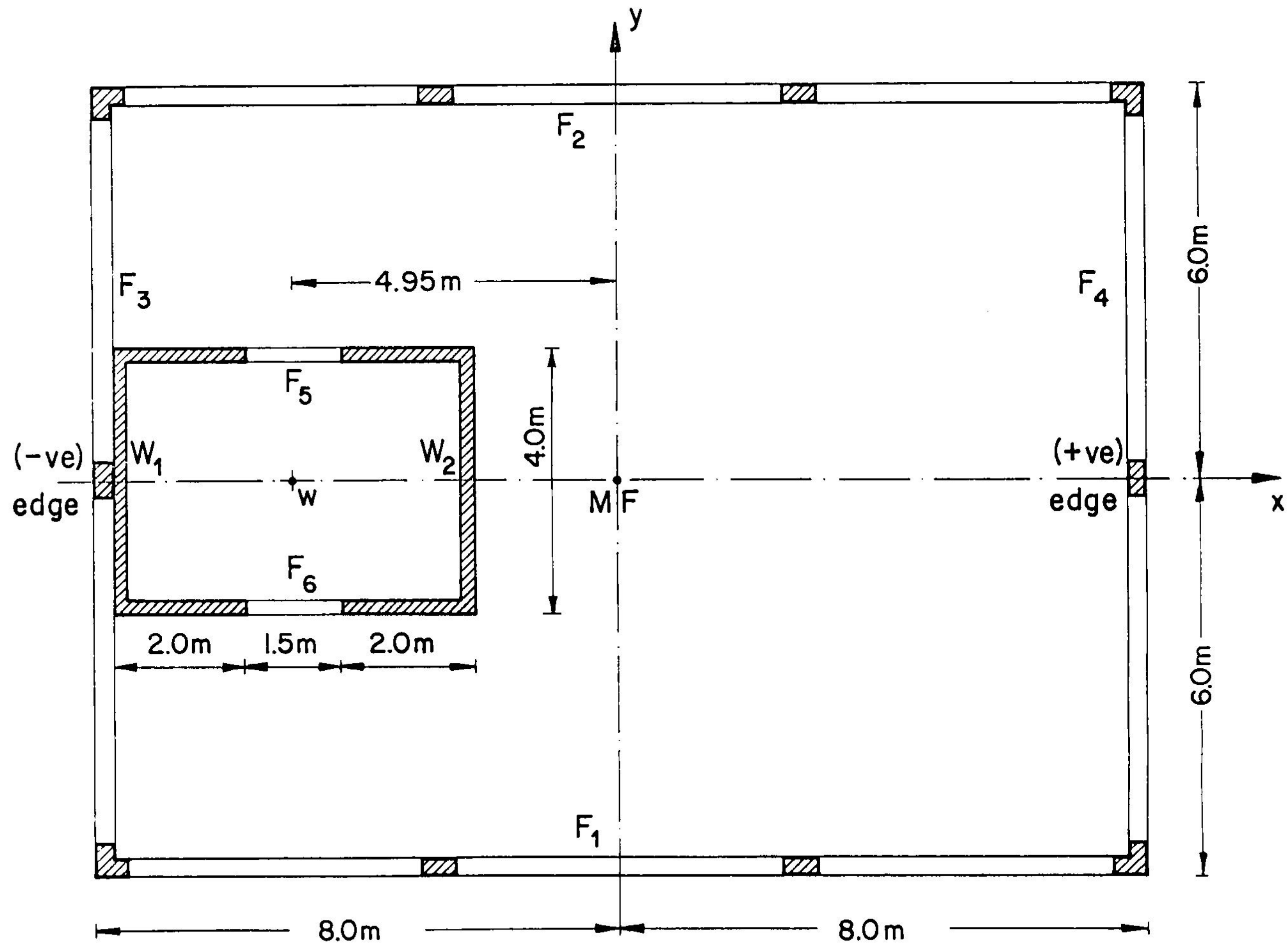

Figure 1 Structural Plan for Example Structure

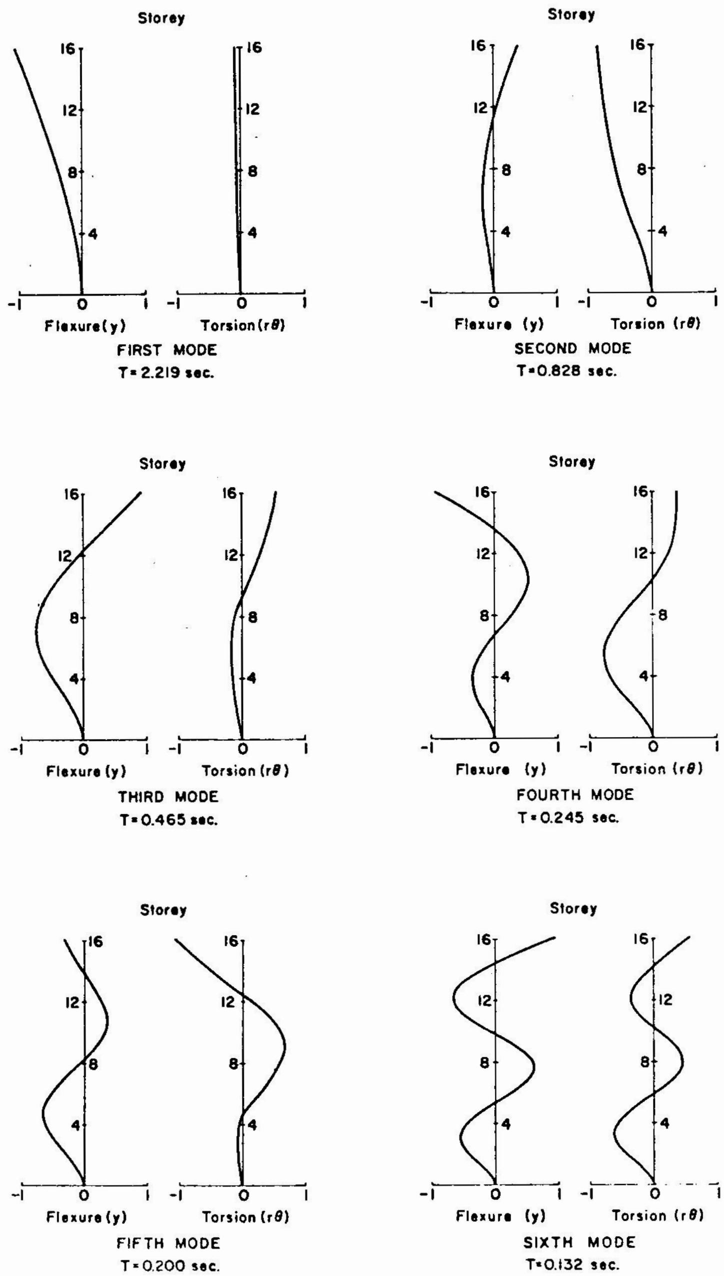

Figure 2 Coupled Flexural-Torsional Mode Shapes for Example Structure

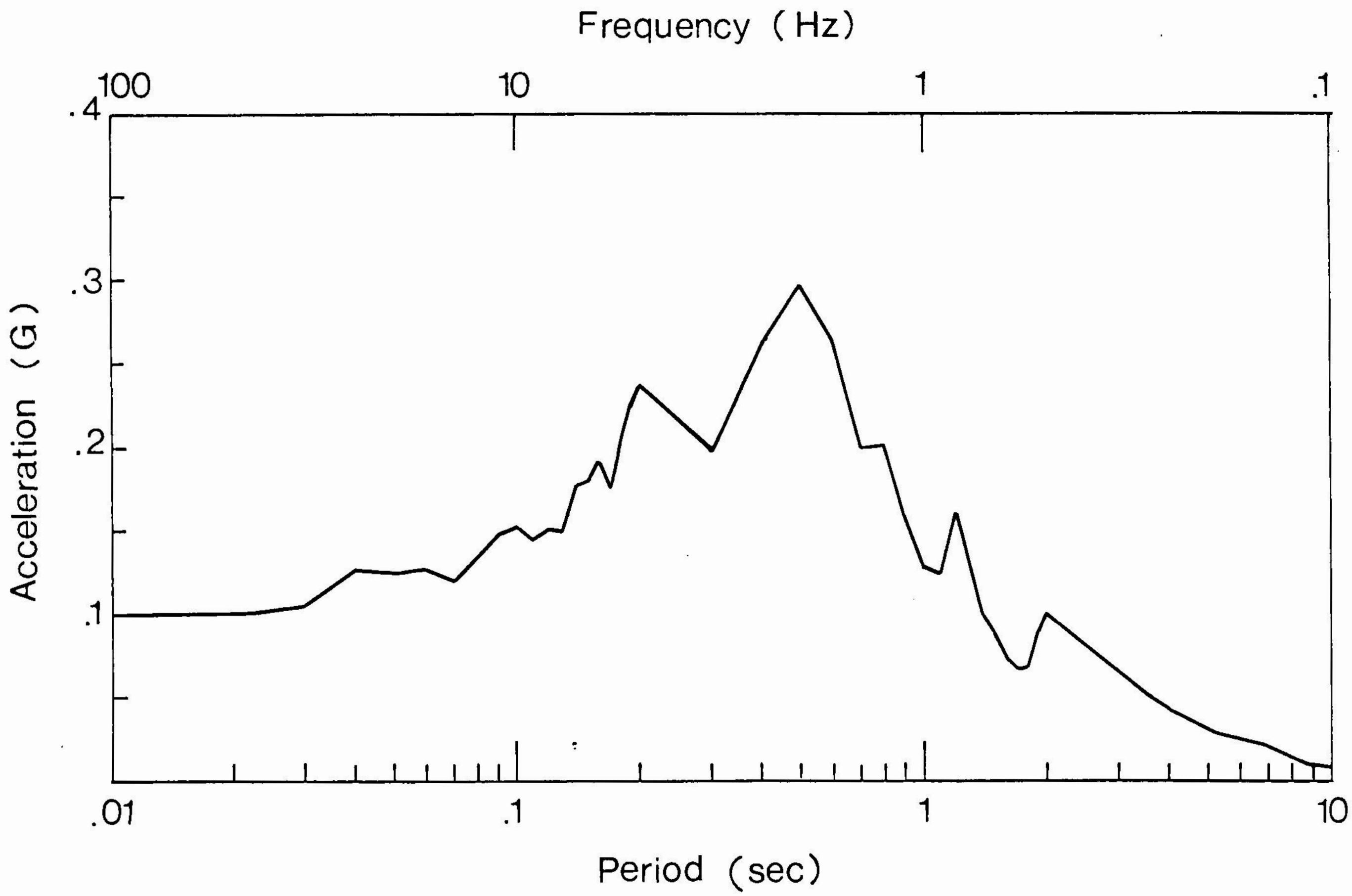

Figure 3 Ground Response Spectrum - El Centro W-E (5% damping)

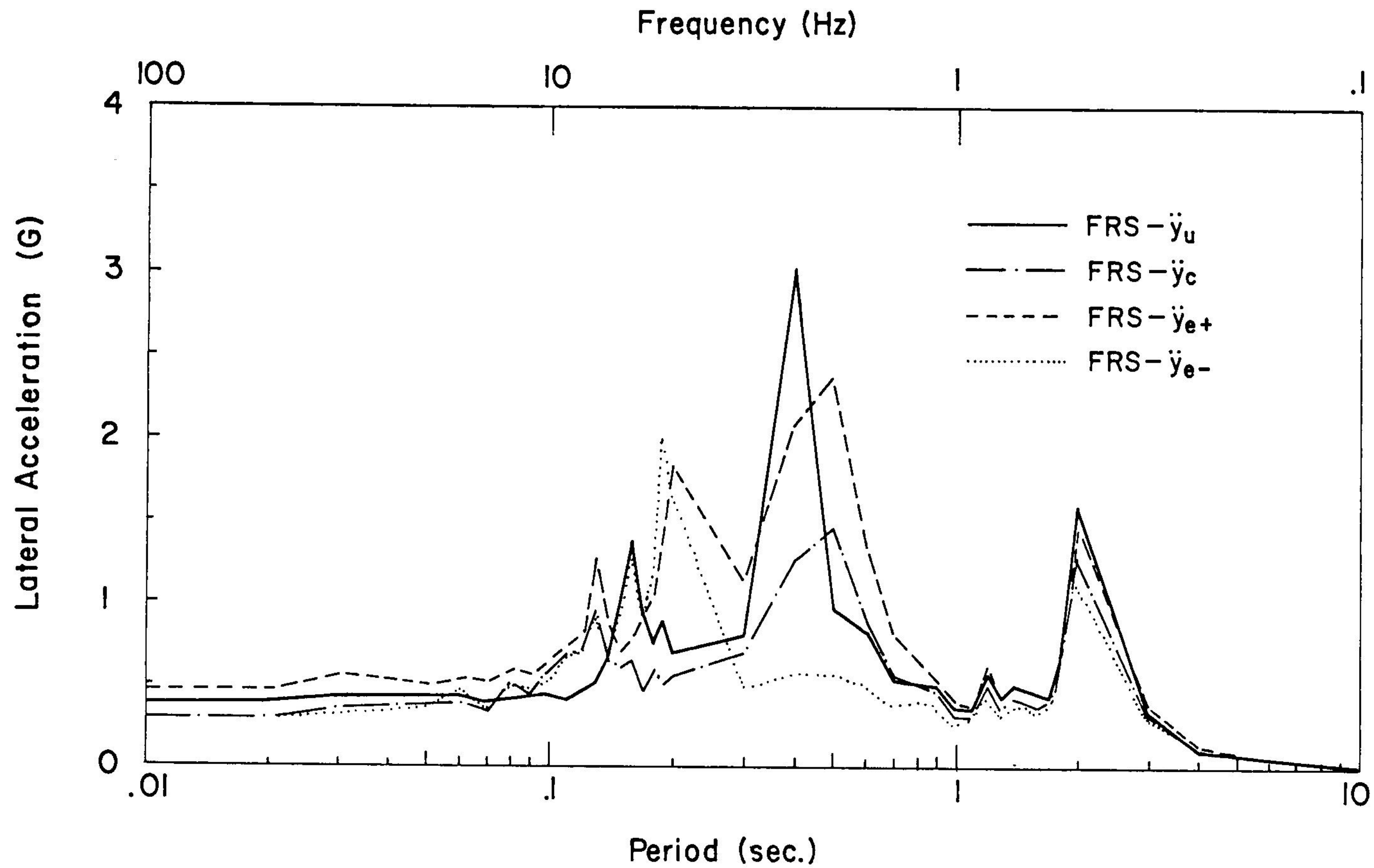

Figure 4 Floor Response Spectra (Building Top)

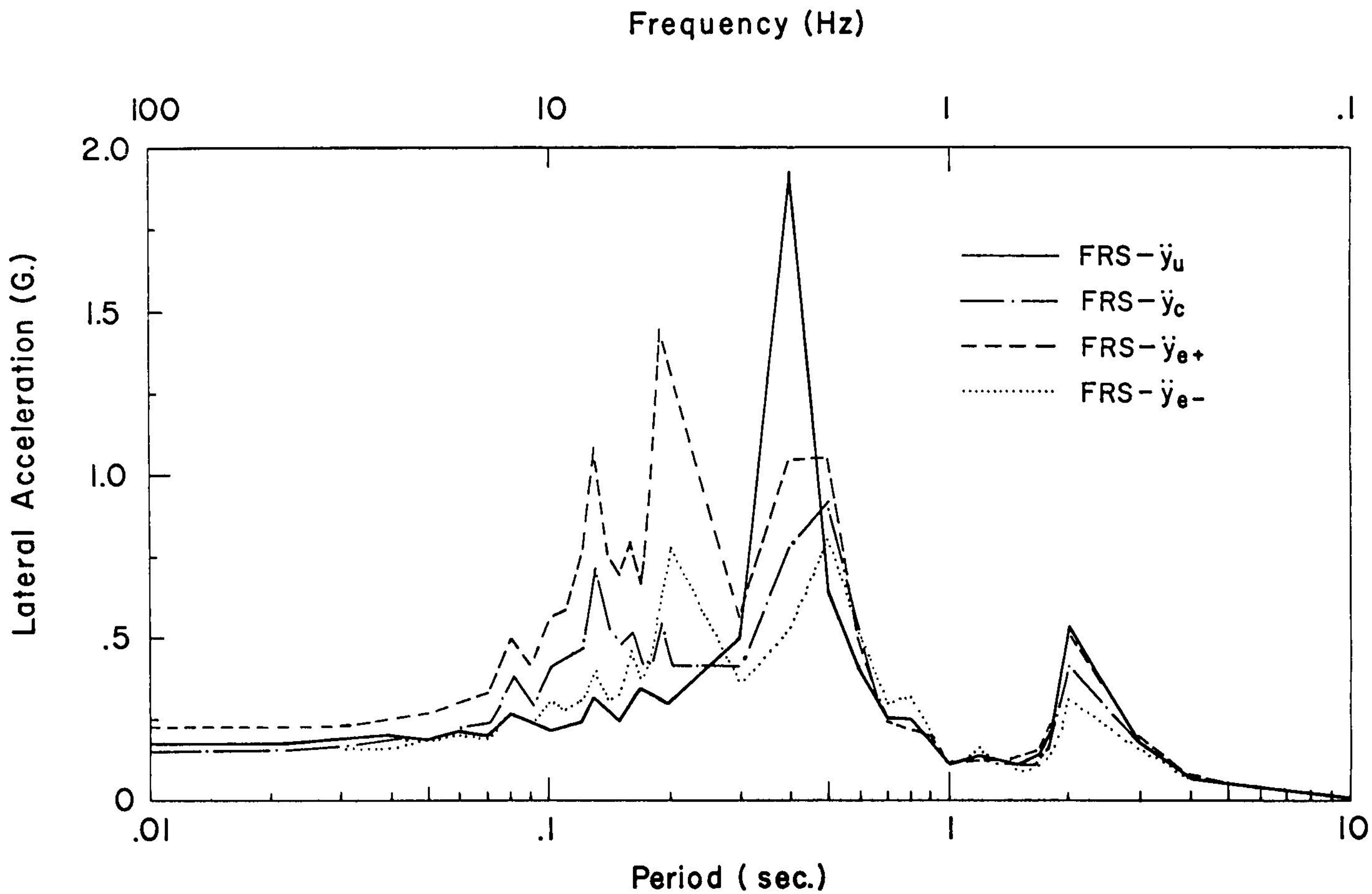

Figure 5 Floor Response Spectra (Building Midheight)

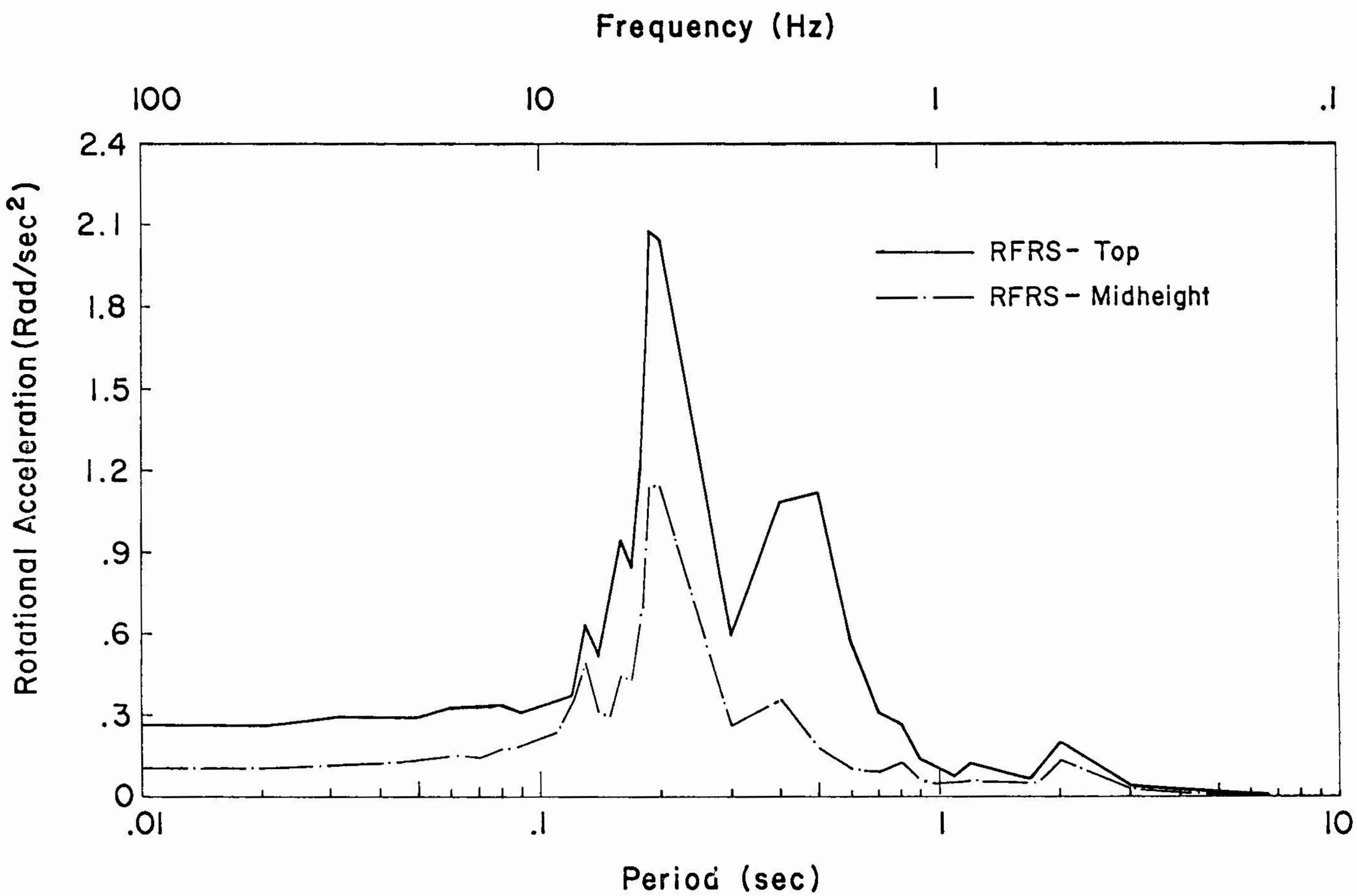

Figure 6 Rotational Floor Response Spectra